$C \subseteq X$,
C is saturated w.r.t. $p: X \to Y$
If $C \cap p^{-1}\{y\} \neq \phi$, $y \in Y$
$\Rightarrow p^{-1}(\{y\}) \subseteq C$.
If $C = p^{-1}(A)$ for some $A \subseteq Y$ then C is saturated w.r.t.

P2

(2) A map $f: X \to Y$ is said to be closed map, if for each closed set A of X, the set $f(A)$ is closed in Y.

* If $p: X \to Y$ is a surjective continuous map, that is either open or closed, then p is a quotient map.

Example(1) :-

Let $X = [0,1] \cup [2,3] \subseteq \mathbb{R}$
$Y = [0,2] \subseteq \mathbb{R}$.

define $p: X \to Y$ by

$$p(x) = \begin{cases} x & ; \text{if } x \in [0,1] \\ x-1 & ; \text{if } x \in [2,3] \end{cases}$$

then p is surjective, continuous and closed
$\therefore$ p is a quotient map.

Let $A = [0,\frac{1}{2}]$ closed in X
$\therefore$ $p(A) = [0,\frac{1}{2}]$ closed in Y.

But P is not an open map.
since $[0,1]$ is open in X $\left\{ [0,1] = X \cap \underbrace{(-1, \tfrac{3}{2})}_{\text{open in}} \subseteq \mathbb{R} \right.$

But $P[0,1] = [0,1]$ not open in Y.
$\Rightarrow$ p is not an open map.

$\to$ If A is a subspace of $[0,1) \cup [2,3]$ of X
i.e If $A = [0,1) \cup [2,3] \subseteq X$.